Is the number of string vacua finite?

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Abstract

Based on work in progress with Bobby Acharya, Rene Reinbacher, Gonzalo Torroba and others.

1. Introduction

Over the past couple of years, as reviewed here by Denef, we have started to shed some light on the distribution of vacua of string/M theory, to get a picture of the "stringy landscape."

What we know so far comes from studying particularly simple limits, obtained by compactifying supergravity on simple target spaces which preserve supersymmetry, and then adding various ingredients (branes, fluxes, and so on) which break it weakly.

Now the ideas of duality suggest that these results are far more broadly applicable. On the other hand, there is also good reason to think that many other constructions and regimes which we do not understand in detail, are not dual to weakly coupled supergravity constructions, or weakly coupled supergravity plus general field theoretic sectors (which we can analyze at strong coupling).

In this sense, much of the landscape is still shrouded in darkness. (Of course, we don't know whether the unknown part is "larger" or "smaller" than the part we can study at present.)

The situation may not be as bad as it sounds. It may be analogous to our understanding of theories with extended supersymmetry c. 1990. Once the ideas of duality were appreciated, it became possible to make simple hypotheses about moduli spaces of vacua, and test them.

What are plausible, simple and testable hypotheses about the entire landscape ?

Probably the most basic question we might ask is,

Is the number of string/M theory vacua finite?

If so, can we estimate this number ? (MRD, talk at JHS60, 2001).

Given finiteness, we can try to ask further questions such as,

- What fraction of vacua are described by weakly coupled construction X ?
- What fraction of vacua have observable property P ?

and so on. As we discussed at previous Strings conferences and review later, this might lead to predictions from string theory.

On the other hand, suppose the number were not finite. At best, we would have to be very careful in our definitions, to characterize the set of vacua.

At worst, we might find an infinite number of different vacua, which agree with known physics (the Standard Model, ...) in detail, and predict any extension we can imagine.

Thus the eventual stakes are rather high.

In fact, it is clear from the start that the number of vacua is not finite (MRD 0303194). The simplest counterexample is $AdS_4 \times S^7$ (Freund and Rubin, 1980). These vacua form a one parameter infinite family, indexed by the flux

$$N = \int_{S^7} *G^{(4)},$$

a positive integer.

Of course, this example is not realistic:

1. It has extended supersymmetry
2. It has no chiral fermions
3. The cosmological constant is comparable to the KK scale.

However, there is no reason to think that remedying these defects would change the underlying reason for the existence of an infinite number of vacua, which is simply the infinite number of possible choices of flux. And, work since 1980 gives ample reason to think that all of these defects can be fixed, say by adding additional branes and sources of vacuum energy.

A more fundamental problem with these vacua is that the radius R of the internal S^7 increases with flux N as

$$R \propto N^{1/6}.$$

Thus, as N becomes large, this series of vacua decompactifies, and all but a finite number of vacua cannot describe our world.

Our question becomes, Is the number of "potentially realistic" vacua finite, one hopes under some simple, easily tested criterion. Large volume is relatively easy to test.

A second class of infinity arises if one allows arbitrarily large supersymmetry breaking. For example, in IIb theory, one can add N space-filling D3 branes and N anti D3-branes, for any N.

While sufficiently large N probably leads to instability, this is not totally obvious. A simpler ground on which to rule them out is that since susy breaking makes a positive contribution to the vacuum energy, all but finitely many of these vacua would be expected to have too much vacuum energy to describe our world. This is another simple criterion.

These considerations motivated the following conjecture (MRD 0303194)

> The predictivity of string/M theory depends on the conjecture that the number of consistent [flux] vacua with cosmological constant $|\Lambda| < \Lambda_{max}$, a bound we choose, and compactification volume $V_M < V^>$, a upper bound, is finite.

At the time, the evidence for this conjecture was rather meager; it was motivated mostly by the hope that string theory will turn out to be predictive (*i.e.*, wishful thinking).

But over the last two years, no clear counterexample has been found, and a fair amount of evidence for it has accumulated, as we now discuss.

2. Many choices

1. Choice of microscopic theory (weakly coupled fundamental degrees of freedom: IIa string, IIb string, het string, M, etc.)
2. Choice of compactification manifold X (or corresponding non-geometric choice, *e.g.* choice of CFT).
3. Choice of bundle or brane configuration
4. Choice of flux
5. Choice of critical point of the effective potential

We all believe choice (1) is finite.

But in no other case is this obvious. (4) is clearly infinite and we require all such series of vacua to run off to large volume or c.c.. (2) is certainly in danger of being infinite. For example, suppose we could use $X \cong X' \times \Sigma$ for Σ a genus g Riemann surface.

Not much prior intuition about (3), (5) or non-geometric aspects of (2).

Finally, there are many dualities/equivalences between these choices and we must be careful not to double count.

3. A simple example

To illustrate some issues, consider the following infinite two parameter family of vacua (Acharya, Denef and Valandro 0502060): Starting from $AdS_4 \times S^7$ with flux N, we quotient S^7 by a freely acting $\mathbb{Z}_k$, to get $AdS_4 \times S^7/\mathbb{Z}_k$.

Now the Einstein equation and thus the relation between flux and metric is locally the same on S^7 and on $S^7/\mathbb{Z}_k$. But, the volume of $S^7/\mathbb{Z}_k$ is $\mathrm{Vol}(S^7)/k$. Thus, we (incorrectly) might expect

$$\mathrm{Vol} \propto \frac{N^{7/6}}{k}.$$

If so, then by simultaneously increasing k and N, we would find an infinite series of vacua with fixed internal volume.

While the above is not quite right, let us discuss some possible ways out if it were, as they make some useful points. As one try, perhaps we need to be more precise about the large volume cutoff. In fact, the phenomenological constraints are of two types.

First, the volume of X enters in the relation between the four and higher dimensional Planck scales,

$$M^2_{Planck\ 4} = \mathrm{Vol}(X)\ M^{D+2}_{Planck\ D+4}$$

where $D = \dim X$. To use this, we work in conventions with fixed $M_{Planck\ 4}$ (since this is observed); then increasing $\mathrm{Vol}(M)$ will decrease the fundamental scale $M_{Planck\ D}$, and eventually we will find light stringy modes or black holes. These would have shown up in collider or cosmic ray experiments, so are constrained to $M > 1$ TeV or so (Anchordoqui et al 0311365).

This gives an upper bound on $\mathrm{Vol}(M)$.

Second, there is a related but different lower bound on the mass of the lightest KK modes, inferred from observation of neutrinos from SN 1987a (Arkani-Hamed et al 9807344).

This is equivalent to an upper bound on the **diameter** d of the internal manifold, the maximal distance between a pair of points

$$d = \max_{x_1,x_2} \text{distance}(x_1, x_2).$$

This is because one can prove that the lowest non-zero eigenvalue of the scalar Laplacian must satisfy

$$M_{KK}^2 \leq \frac{\pi^2}{d^2},$$

as one can see by considering a trial wave function

$$\psi(x) = \cos \frac{\pi d(x_1, x)}{d(x_1, x_2)}.$$

Since powers $(\psi(x))^n$ give approximate eigenfunctions with $M \sim nM_{KK}$, this gives rise to a tower of KK modes.

In our example, $S^7/\mathbb{Z}_k$ can be anisotropic, so that k does not enter the diameter. Then, if we stay in the controlled regime $\mathrm{Vol} >> 1/M_{pl}^7$, the diameter runs off to infinity as

$$d \propto N^{1/6}.$$

Thus, we would save the conjecture by insisting that diameter instead of volume remain bounded. In general, we could enforce bounds on both.

A variation on this would be a series of models with a tower of light wrapped brane states. For example, in string theory, we might have winding strings T-dual to the KK towers.

Note that both diameter and volume control four dimensional physical observables, the KK scale and the scale of higher dimensional stringy or quantum gravity effects.

Using these as the definitions, we could enforce similar bounds in non-geometric compactifications.

Actually, the above is **not** the resolution in this particular example. Rather, in going to $S^7/\mathbb{Z}_k$, the flux quantization condition is modified, to integral quantization of

$$N' = \int_{S^7/\mathbb{Z}_k} *G^{(4)}.$$

Relating this to the definition of flux N on S^7, we have

$$N = kN',$$

so in terms of the integral quantized N' we have

$$\text{Vol} \propto (N')^{7/6}\ k^{1/6}$$

which will run off to infinity for any infinite sequence.

In this ensemble, the number of vacua with volume less than V_* goes as

$$N[\text{Vol} \le V_*] \sim V_*^6,$$

a rather fast growth. We will return to this.

But for now, our main point is that by combining various choices, one gets new infinite series. It is not obvious that these always run off to large V or Λ. But it is true in the many examples we have checked so far, such as $Y^{p,q}$ spaces.

4. Choice of bundle

Given X, we need to choose an $E_8 \times E_8$ or $SO(32)$ bundle (heterotic and type I), embeddings of D7/D3-branes and bundles (IIb), D6-branes (IIa), or singularities and bundles (M theory). These constructions are all related by known dualities, but details are not totally under control yet.

One necessary ingredient for finiteness is the anomaly cancellation or tadpole conditions. In the perturbative heterotic string, one requires a bundle with $c_2(V) = c_2(TX)$. There is then a theorem (Maruyama 1981) that the moduli space of these is algebraic (has finitely many branches). This can be verified in elliptic fibration constructions and explicit bounds on $c_3(V)$ obtained (work with Reinbacher).

More generally, one has

$$c_2(V) + N_{5B} = c_2(TX)$$

and one needs to use the fact that stable bundles have lower bounds on $c_2(V)$ (the Bogomolov inequality). Actually, the known lower bound is on $c_2(V) \wedge J$, leading to subtleties but no clear counterexample.

Of course, one can also add anti 5B's. Eventually one needs to call on instability or an upper bound on the vacuum energy.

While brane constructions are quite analogous and similar claims will hold at large volume, this does not appear to be true in general, in part because one can turn on $U(1)$ magnetic fluxes, which cancel out of the tadpole conditions. To see this, note that the condition for supersymmetry (in type I and IIb) is that the complex numbers

$$\mathrm{Tr}\ (B - F + iJ)^{p/2}$$

on every brane have the same phase. This does not force the Chern characters $\mathrm{Tr}\ F^k$ to take a definite sign.

Antoniadis, Kumar and Maillard 0412008, 0505260 have used this to find an infinite series of type I vacua on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$. However, the supersymmetry conditions force some Kähler moduli off to infinite volume.

Note that (0505260) claims to find infinite series which stay at finite volume. However, I believe these have not taken flux quantization into account properly.

In (Blumenhagen et al 0411173), analyzing statistics of IIa branes, an argument is given that at fixed complex structure, finitely many susy brane configurations are possible. After T-duality this is consistent with the above.

J. Kumar and J. Wells point out that one can find combinations of branes, each supersymmetric at different points in moduli space, which together have negative tadpole (*i.e.* it can be cancelled by susy branes and flux). If these could recombine to a single susy brane, one would find infinitely many vacua.

5. Choice of minimum

Could a single effective potential have infinitely many minima?

In principle, why not – consider an electron in an infinite crystal.

Now physically distinct vacua must be separated in configuration space, and consistent with this, the crystal has infinite volume. A simple argument that this could not happen in string theory, would be if the relevant configuration spaces have finite volume (after duality equivalences).

Conversely, if any "naturally occuring" moduli space in supersymmetric theories has infinite volume, this is a reason to worry (related but different concerns in (Horne and Moore 9403058)).

Conjecture: All physical moduli spaces are finite volume in the physical metric (after imposing duality equivalences):

- Conformal field theories, in the Zamolodchikov metric
- Non-conformal field theories (likewise). Typically flows to IR go to stronger coupling and thus decrease the Zamolodchikov volume.
- etc...

Obviously very far reaching. I only know one counterexample: the single free real boson, whose single modulus is the radius R. The distance to $R = \infty$ is infinite. Of course if we use this to define a string theory world-sheet, this is "large volume," but one might worry that it appears in other ways. In any case, sigma model moduli spaces for higher dimensional tori, Calabi-Yau's etc. are finite volume.

I would not be totally surprised to hear of other counterexamples, and then one would have to think about this.

6. Choice of flux

Most compactifications allow far more fluxes than Freund-Rubin. And, turning on fluxes on conjugate cycles (*e.g.* intersecting Σ_A and Σ_B) typically leads to potentials that stabilize moduli in the interior of moduli space, e.g.

$$W = \int F^{(2)} \wedge J \wedge J + F^{(4)} \wedge J$$

would stabilize $J \sim F^{(4)}/F^{(2)}$.

This potential infinity can be removed by a combination of duality equivalences, and constraints on the flux, e.g. the IIb tadpole constraint

$$\int F^{(3)} \wedge H^{(3)} + N_{D3} = \frac{\chi}{24}$$

The detailed discussion is rather intricate but can be summarized in flux counting formulas such as (Ashok-Douglas 0307049),

$$N_{vac} = \frac{(2\pi L)^{b_3}}{b_3! \pi^m} \int \det(R + \omega \cdot \mathbf{1}).$$

This reduces the finiteness question to finiteness of the integral of $\det(R+\omega)$, modulo one further loophole (below). Unfortunately it is not known whether this is finite in general. But since $\int \det \omega$ is finite, and R is bounded away from singularities, it could only diverge at singularities. So far this has been checked explicitly only for the conifold point. With G. Torroba, we are checking it for the next simplest singularity, of the form $xy = w^2 + z^3$, *i.e.* the $\mathbf{Z}_3$ "Argyres-Douglas" point.

The loophole allows infinite series of vacua coming from sequences of fluxes like

$$F = (F_{A1}, F_{A2}, \ldots, F_{An} = 0, F_{B1}, \ldots, F_{Bn})$$
$$H = (H_{A1}, H_{A2}, \ldots, H_{An}, H_{B1}, \ldots, H_{Bn}).$$

Since $F_{An} = 0$, H_{An} does not appear in the constraint $F \wedge H$, and can be arbitrarily large. However, general arguments in (0307049) show that these must run off to a degeneration limit, of which the only known examples are large complex structure (IIb) and the mirror large volume (IIa).

Indeed, such a series appeared in $K3 \times T^2$ compactification (Tripathy and Trivedi 0301139). The IIb examples are formally much like those of (De Wolfe et al 0505160) but appear to have $e^K|W|^2 \sim 1$, so may not be physical (?).

7. Choice of manifold

It is not obvious that the number of (topologically distinct) usable compactification manifolds X is finite. Mathematicians are not even sure that the number of compact Calabi-Yau threefolds is finite.

In fact, existing mathematical results in comparison geometry allow one to argue that, even if the number of possible compactification manifolds is infinite, this would not invalidate the physical finiteness conjecture.

Theorem (Cheeger, 1979). A set of manifolds with metrics $\{X_i\}$, satisfying the following bounds,

- $\text{diameter}(X_i) < d_{max}$
- $\text{Vol}(X_i) > V_{min}$
- Curvature K satisfies $|K(X_i)| < K_{max}$ at every point,

contains a finite number of distinct homeomorphism types (and diffeomorphism types in $D \neq 4$).

By curvature K, we mean all sectional curvatures (i.e. the curvature scalar in every two-dimensional subplane).

For present purposes, we can take

- $\mathrm{diameter}(M_i) < d_{max}$ the phenomenological bound,
- $\mathrm{Vol}(M_i) > V_{min} = 1/M_{pl}^D$ the fundamental scale, and
- $|K(M_i)| < K_{max} = \min 1/M_{pl}^2, 1/M_s^2$ likewise.

If we violate the last two bounds, we are out of the regime of conventional geometry, and cannot expect a discussion based on supergravity and conventional metrics to apply.

Thus, even if there were an infinite number of topologically distinct CY's, any infinite series of controlled supergravity compactifications using them would run off to large diameter and violate the phenomenological bound.

There are several variations on this theorem with different assumptions.

The idea behind Cheeger's theorem is that the bounds on volume and curvature are used to show that there exists some $r > 0$ (the injectivity radius), such that X can be covered by contractible balls of radius r.

Then, given the upper bound on diameter, one shows that there is a maximum number N_{max} of balls which suffice to cover X.

Finally, one counts the number of topologically distinct ways to glue N balls together.

So at least for this part of the problem, we have a reason to expect finiteness. Of course a comparable discussion taking into account all the choices in string theory would require many new ingredients, most of which are not yet mathematically precise. But perhaps progress can be made in this direction.

8. Is finiteness enough?

Obviously not, if the number of vacua is too large. A very rough estimate of "too large" is, far larger than 10^{240}, the "inverse volume of the Standard Model."

An example to make this point is to ask, suppose we observed cosmological time variation of the fine structure constant (e.g. see (Tzanavaris, Webb et al, astro-ph/0412649). Such a time variation requires the effective potential $V(\alpha)$ to be very flat. But, as argued in (Banks et al 0112059), known QED corrections make $V(\alpha)$ depend strongly on α, so such a variation would be "highly unnatural" in the standard theoretical sense – it would require tuning at least 8 terms in the Taylor series of $V(\alpha)$ to an approximately 10^{-60} precision.

On the other hand, if these couplings were uniformly distributed, and we had many more than $10^{480+240}$ otherwise acceptable vacua, we could not confidently claim that this observation falsified string theory.

In general, as N_{vac} becomes large, we start to lose any hope of falsifying the theory in this way. Of course, we might still find direct evidence for string theory. The point is to find out what type of game we are playing.

As discussed by Denef, while IIb compactifications may not be too many, there might be 10^{5000} F theory vacua. Of course we have the standard caveats that taking into account $\Lambda > 0$, metastability, dualities and other constraints might drastically reduce these numbers.

Still, these numbers give us strong motivation to look for additional selection principles, such as cosmological selection. Perhaps most of these putative vacua are heavily "disfavored" in some way.

In addition, as we mentioned and as discussed here by Denef and Kachru, there is now good evidence that there are series of vacua for which the number grows with volume,

- $dN \sim d(V^6)$ for M theory flux vacua
- $dN \sim d(V^{2n/3})$ for IIa flux vacua

While these series fit with the finiteness conjecture, if there are otherwise phenomenologically acceptable families of this type, they could lead to another problem. Namely, if the distribution of vacua $N(\mathrm{Vol})$ is highly peaked just below the phenomenological cutoff, then one might be led to "predict" large extra dimensions will be discovered at (say) LHC.

This sounds highly suspicious – why is Vol not even larger, and already seen experimentally? There is no clear anthropic or environmental argument why they could not be.

One valid response to this criticism is that we need to distinguish different senses of "prediction." If, at the end of our analysis, with all consistency conditions and other constraints taken into account, we really decide there are (say) 10^{20} "large volume" vacua, and 10^{-20} "small volume" vacua, then we would have good reason to assert that string theory predicts large extra dimensions, in the strong sense that their non-observation is evidence against the theory.

But if there are many vacua of both types, we will not get this type of prediction. The existence of many large volume vacua does not in itself remove the small volume candidate vacua, nor does it make them "less likely," whatever that means.

It is not yet clear that the M theory or IIa flux vacua are phenomenologically viable. A very interesting discovery of (De Wolfe et al 0505160) is that in the large volume IIa flux vacua, the string coupling also goes to zero as

$$g_s \sim \frac{1}{\mathrm{Vol}^{1/2}}.$$

On the other hand, to get a phenomenologically viable model, one needs $\alpha_{YM} \sim 1/25$. While the small g_s might be compensated by wrapping the brane on a very small cycle, stabilizing the complex structure at such extreme values sounds highly unnatural (since the complex modulus stabilization in IIa is highly constrained).

The M theory vacua do not have a small g_s, but since the KK scale is not parameterically small, are probably hard to stabilize after supersymmetry breaking (?)

Still, I see no reason that a phenomenologically viable series of large volume vacua could not exist.

Thus, I feel a better response to the situation is to take it as motivation to find an additional selection effect which disfavors large volume. As yet there is no clear reason for such vacua to be less stable than others (but it is clear that potential barriers decrease with volume, so perhaps they are). But it seems *a priori* plausible that cosmological selection could depend on the volume of the extra dimensions.

Even a number distribution $V^{2n/3}$ with n large, would be rapidly dominated by a suppression $\exp -V$. This would even peak at $V \sim n/M_F^D$, so might even help in explaining the hierarchy between $M_{Planck\ 4} \sim 10^{19}$ GeV and $M_{GUT} \sim 10^{16}$ GeV.

In any case, the existence of such selection effects is logically independent of whether large volume vacua exist are or phenomenologically viable, and must be argued separately.

9. Vacuum selection

There are various ways in which "vacua" according to the analysis so far might be favored or disfavored. We can distinguish various "levels" at which this might happen. Ordered by "time," these are

H	initial wave function (Hartle-Hawking, etc.)
E	quantum cosmological dynamics (eternal inflation)
D	semiclassical cosmological dynamics (slow roll inflation, descent to minimum)
C	consistency and metastability
A	phenomenologically (or anthropically) acceptable

A cosmological measure on vacua must incorporate all of these effects, and perhaps others. This topic deserves a talk of its own.

While it is tempting to say that the "higher" levels are more fundamental, it should be realized that it is very easy for "lower" levels to wash out (make irrelevant) the previous structure in the measure. For example, eternal inflation could make the initial wave function irrelevant.

The wave function and eternal inflation ideas for selection principles have been studied for over 20 years and remain problematic.

On the other hand, we do believe that our universe went through a period of slow-roll inflation caused by a vacuum energy somewhere between

$$(1 \text{ TeV})^4 \leq \Lambda \leq (10^{15} \text{ GeV})^4,$$

before rolling down to its present minimum.

Can we derive any selection principles from this? In other words, are there some vacua which are extremely unlikely to arise this way?

An argument that large extra dimensions are extremely unlikely to arise:

1. Vacuum energy in a flux vacuum falls off with volume, in IIb as $\Lambda \sim 1/(\mathrm{Vol})^2$, and in IIa even faster. Thus, a lower bound on the scale of inflation, provides an upper bound on the volume of the extra dimensions during inflation.

2. Furthermore, slow roll inflation requires that this volume be approximately stabilized, so the flux must be small.

3. Flux can then change after inflation, but only by tunneling.

4. A tunnelling event which changes flux by a large amount ΔN is highly suppressed, by order $\exp -\Delta N \sim \exp -V^{2/3}$ in IIa theory.

5. One might avoid this by a succession of tunnelling events with small ΔN, but this process takes so long that all matter is inflated away, and one is left with an empty universe. (Brown-Teitelboim 1988, Bousso-Polchinski 0004134)

This seems reasonable but does not in itself rule out large extra dimensions; rather it says that they are only consistent with a very low scale of inflation. We might get observational constraints on this scale, or we might appeal to a selection principle such as that discussed here by Tye to favor high scales.

10. Conclusions and new directions

- For many reasons, we would like a general picture of how the vacua of string/M theory populate the space of possible low energy physical theories.

- The simplest question one can ask is whether and in what sense these vacua are finite. If so, one can go on to estimate their number. If not, one will worry that they densely populate the space of physical possibilities, until argued otherwise.

- The standing conjecture in this area is that the number of vacua satisfying a given upper bound on the volume of the extra dimensions is finite. For this to be true, one must take into account expected instabilities in vacua with large supersymmetry breaking. This can be crudely modelled by putting an upper bound on the vacuum energy as well.

- It is useful to refine this conjecture by distinguishing the diameter, related to the KK scale, and the volume, related to the fundamental scale, and placing upper bounds on both. Since they directly control physical observables, one can define analogs of these quantities in non-geometric compactifications.

Having looked at many cases, there is no clear counterexample (an infinite series not running off to large volume), but there are cases not yet settled. Perhaps the most likely sources of counterexamples at this point are

- Infinite volume moduli spaces.
- Singularities at which $\det R$ is not integrable, at which vacua would accumulate,

But no clear argument has emerged that supersymmetric brane configurations are always finite in number, nor for finiteness in many other cases.

A possible model for conceptual understanding of this point are the finiteness theorems of comparison geometry, such as Cheeger's theorem. We used this to argue that, even if there are infinite numbers of (say) Calabi-Yau's, the corresponding infinite sequence of vacua will run off to large diameter.

The basic idea is that bounds on curvature allow us to build any manifold of finite size out of a finite number of contractible balls; there are then finitely many ways to attach these together. While based on conventional Riemannian geometry, only very elementary properties are used, so it might be possible to generalize to string/M theoretic geometries.

There is increasing evidence that infinite series of phenomenologically viable vacua running off to large volume could exist. If so, this will raise a "large volume problem" for string theory, that we will not have a natural explanation why our vacuum is compact with a definite volume.

In itself this does not raise the spectre of non-falsifiability, but it deserves to be addressed, perhaps by finding a cosmological selection argument which disfavors large volume. Of course, the discovery of large extra dimensions would be very dramatic and welcome for string theorists, so we should not argue too strenuously that they are disfavored (unless they are).

To conclude, the present status is that the various distributions of vacua which arise in different constructions – IIb, M theory, IIa, heterotic – are similar but disagree in detail. This suggests that none of them are truly representative of the totality of string/M theory vacua.

I think the way to progress on this is to try to make our qualitative pictures of duality between these constructions more precise, by postulating simple model ensembles which incorporate all of the choices suggested by all of the dual constructions, and try to interpolate between the various weakly coupled regimes. These could then be tested in ways analogous to those used in previous studies of duality.

Some simple steps in this direction have been taken in IIa-IIb orientifold constructions (Derendinger et al 0503229; Villadoro and Zwirner, 0503169; Cámara et al 0506066; Berglund and Mayr 0504058; Louis et al) and much current work in generalized complex geometry and non-geometric compactification is also very relevant for this. I am hopeful that we will see significant progress here by Strings 2006.