Interfacial Behaviour in Tunnel Engineering

Industrial Presenter: Zongxi Cai, Northern Heavy Industries Group Co., Ltd.

Participants: C. Sean Bohun, Hui-Hui Dai, Justin Findlay, Matt Hennessy, Peter Howell, Huaxiong Huang, Greg Lewis, Roderick Melnik, Richard Moore, Robert Miura, John Ockendon, Yonji Tan, Burt Tilley, Jonathan Wylie

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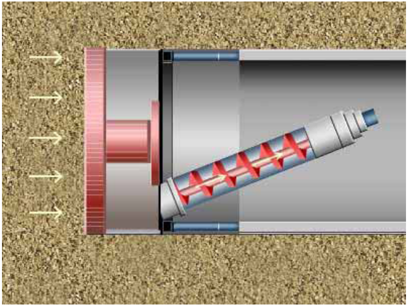
Outline

The Tunnel Borning Machine (TBM)



- 4-80m below ground surface
- 18m in diameter and 400m long
- "mucking speed" at about 500m/month
- pressure is applied at cutting head to stabilize excavation face
- removal of excavated material is controlled

Collapse Potential

- issues when applied pressure or mucking speed is too high or too low
  - for soil, instability of excavation face can lead to collapsed ground
  - for rock, induced shocks can damage machine
- surface displacements of 1cm to 3cm can cause damage to buildings

NHI Goals for the Workshop

- Main Goals
  - Determine the best constitutive model for the analysis of the interfacial behaviour
  - Determine the most important mechanisms that lead to instability of the excavation face
  -
- Specific Goals
  - provide information about how to prevent ground surface collapse
  - provide information about how to prevent ground surface uplift
  - determine the loadings acting on a single cutter
  - determine the distribution of the interfacial force between the cutting head and formation

Key Observations

- TBM is pressurised during regular operation
  - Soil is not hard ($E \ll 60\,\mathrm{MPa}$). $\Rightarrow$ significant plastic region.
  - Hydrostatic stresses in soil stabilize matrix, anticipate plastic region has finite extent.
  - Competition between loading pressure/shear stress along drill bit interface and hydrostatic stresses.

Two modeling approaches:

Plastic/Elastic: Plastic region in elastic medium
Plastic/Rigid: Plastic region in rigid medium

Mohr-Coulomb Criteria

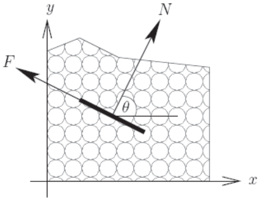


Figure: Slip surface with force balance in static granular material. (Howell and Ockendon, *to appear*).

- N is normal force on particles below the surface element due to particles above
- F is frictional force on surface element
- Coulomb's law states

$$|F| \leq |N| \tan \phi \qquad (1)$$

where ϕ is a material parameter called the "angle of friction"

Mohr-Coulomb Criteria

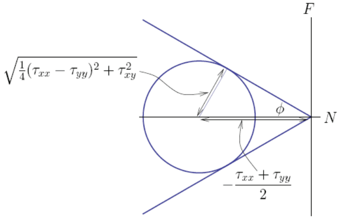


Figure: The Mohr circle in the (N, F)-plane, where the lines represent $|F| = |N| \tan\phi$ (Howell and Ockendon, *to appear*).

- material flows when Mohr-Coulomb's yield criteria is satisfied

$$2\left(\sigma_{xx}\sigma_{yy}-\sigma_{xy}^2\right)^{1/2} = -\left(\sigma_{xx}+\sigma_{yy}\right)\cos\phi \qquad (2)$$

General Mohr-Coulomb Criteria

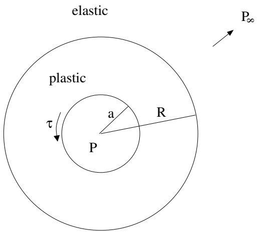
$$C \geq \frac{1}{2}\left[(\sigma_{xx} - \sigma_{yy})^2 + 4\sigma_{xy}^2\right]^{1/2} + \sin\phi\,(\sigma_{xx} + \sigma_{yy}) = f(\sigma_{ij}) \quad (3)$$

where C is the “cohesion”

- material is solid when (strict) inequality holds
- material is plastic when the equality holds

Outline

Axisymmetric Cross-Section Problem



- Stresses independent of θ
- Mohr-Coulomb yield criteria defines elastic-plastic boundary
- Stress balance (axisymmetric)

$$\frac{1}{r}\frac{d}{dr}(r\sigma_{rr}) - \frac{1}{r}\sigma_{\theta\theta} = 0 \quad (4)$$

$$\frac{1}{r}\frac{d}{dr}(r\sigma_{r\theta}) + \frac{1}{r}\sigma_{r\theta} = 0 \quad (5)$$

Boundary Conditions

- $\sigma_{rr} \to -P_\infty$ as $r \to \infty$
- $\sigma_{r\theta} = \tau$ at $r = a$, where τ is applied stress due to boring
- $\sigma_{rr} = -P$ at $r = a$, where P is applied pressure
- At elastic-plastic boundary $r \equiv R$
  - the Mohr-Coulomb criteria is satisfied
  - $\underline{\sigma} \cdot e_r$ continuous

    $\to$ σ_{rr} and $\sigma_{r\theta}$ is continuous

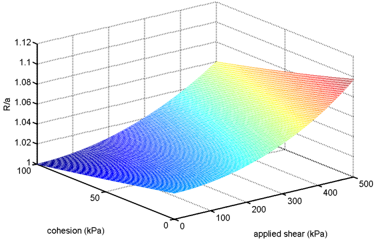
    $\to$ $\sigma_{\theta\theta}$ also continuous

Results-No Cohesion

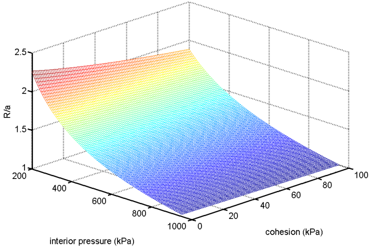
Assuming $C = 0$ get (where $v = \sigma_{rr}/R^2$ and $\hat{R} = R/a$):

$$2 \ln \hat{R} = \int_{-P}^{\hat{R}^2\left(-P_\infty + \sqrt{P_\infty^2 \sin^2\phi - \tau^2/\hat{R}^4}\right)} \left\{ \frac{\cos^2\phi}{v - \sqrt{v^2 \sin^2\phi - \tau^2 \cos^2\phi}} \right\} dv \qquad (6)$$

Results-Cohesion



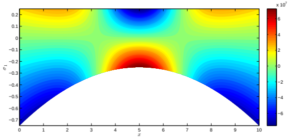
$\phi = 30^\circ, p = 2\text{MPa}$

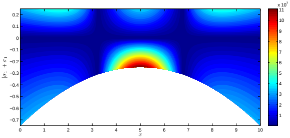


$\phi = 30^\circ, \tau = 64\text{kPa}$

- Damage radius more sensitive to variations in applied pressure

Density-Driven Instability





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Equations of Plasticity

$$\nabla \cdot \boldsymbol{\sigma} = \rho \mathbf{g} \ , \ \Lambda \dot{e}_{ij} = \frac{\partial f}{\partial \sigma_{ij}} \ , \ f(\sigma_{ij}) = 0 \tag{7}$$

$$\dot{e}_{ij} = \frac{1}{2} \left\{ \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right\} \tag{8}$$

- 10 equations for 10 unknowns
- Start with the simplest configuration:
  - No gravity
  - Point-force in x-direction.

2D problem

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = -F\delta(x)\,\delta(y) \qquad (9)$$

$$\frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{yy}}{\partial y} = 0 \qquad (10)$$

$$\sqrt{(\sigma_{xx} - \sigma_{yy})^2 + 4\sigma_{xy}^2} + \sin\phi\,(\sigma_{xx} + \sigma_{yy}) = 0 \qquad (11)$$

- 3 equations and 3 unknowns
- hyperbolic in stress

Granular material in a hopper

From Brennan & Pearce (1978), Tayler (2002)

Let

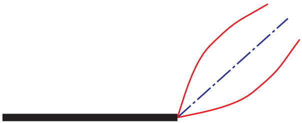
$$p = -\frac{\sigma_{xx} + \sigma_{yy}}{2} \; , \; \tan 2\psi = \frac{2\sigma_{xy}}{\sigma_{xx} - \sigma_{yy}} \tag{12}$$

$$A \begin{pmatrix} p \\ \psi \end{pmatrix}_x + B \begin{pmatrix} p \\ \psi \end{pmatrix}_y = \begin{pmatrix} 0 \\ \csc\phi \end{pmatrix} \tag{13}$$

where Characteristics and Riemann invariants

$$\lambda = \tan\left(\psi + \frac{\pi}{2} \pm \left(\frac{\pi}{4} + \frac{\phi}{2}\right)\right) \; , \; \pm \cot\phi \, \log p + 2\psi = \text{const} \; . \tag{14}$$

Slip lines



- Causality: what initial data is needed that satisfies the conditions in practice?

Similarity Solution

Stress scales like r^{-1}:

$$\sigma_{\theta\theta} = \frac{A\cos\theta}{r} \ , \ \sigma_{r\theta} = \frac{A\sin\theta}{r} \ ,$$

$$\sigma_{rr} = \frac{A}{r}\left\{\cos\theta\left(2\sec^2\phi - 1\right) + \sec^2\phi\sqrt{2\left(\cos 2\theta - \cos 2\phi\right)}\right\} \ .$$

Flow Rule: $\dot{e}_{ij} = \Lambda \frac{\partial f}{\partial \sigma_{ij}}$,

$$\dot{e}_{rr} = \frac{\partial v_r}{\partial r} = \Lambda f_{rr}(\theta,\phi) \quad (15)$$

$$\dot{e}_{r\theta} = \frac{1}{r}\frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} = 2\Lambda f_{r\theta}(\theta,\phi) \quad (16)$$

$$\dot{e}_{\theta\theta} = \frac{1}{r}\left(\frac{\partial v_\theta}{\partial \theta} + v_r\right) = \Lambda f_{\theta\theta}(\theta,\phi) \ , \quad (17)$$

Similarity form:

$$\Lambda = \lambda(\theta) r^{\alpha} \ , \ v_r = w_r(\theta) r^{\alpha+1} \ , \ v_\theta = w_\theta(\theta) r^{\alpha+1} \ ,$$

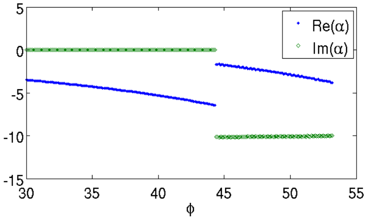
Linear Nonstandard Eigenvalue Problem

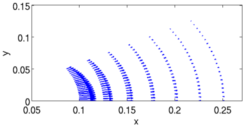
$$\frac{d}{d\theta}\begin{pmatrix} w_\theta \\ w_r \end{pmatrix} = \begin{pmatrix} 0 & \frac{\alpha f_{\theta\theta}}{f_{rr}} - 1 \\ (1-\alpha) & \frac{\alpha f_{r\theta}}{f_{rr}} \end{pmatrix} \begin{pmatrix} w_\theta \\ w_r \end{pmatrix} \text{ on } \theta \in (0,\phi) \tag{18}$$

with

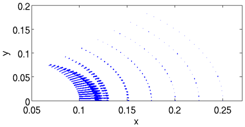
$$w_\theta(0) = w_\theta(\phi) = 0 \ .$$

Similarity Solution

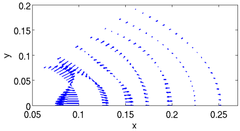




$\phi = 30°$



$\phi = 40°$



$\phi = 50°$

Tidying-Up List

- Include gravity: If small, could match to far-field behaviour
- Free surface (ground): Need to include a free-surface theory.
- Find how stress amplitude A depends on force amplitude F.
- Consider the axisymmetric problem for F
- Point-moment problem at $x = y = 0$.
- May need to regularise rigid region with elastic region for $E \gg 1$

Outline

Conclusions

- Damage region sensitive to applied pressure
  - Axisymmetric model: cohesion reduces damage region, shear increases region.
  - Bridge problem: Damage region less dense, hydrostatic stresses can induce plasticity/gravity instability.
  - Similarity solution: Different solutions depending on amount of internal friction in the matrix

Understanding material properties of soil prior to tunnelling critical in formation of plastic regions based on these geometrically simplified situations.