THE FIELDS INSTITUTE
FOR RESEARCH IN MATHEMATICAL SCIENCES

ABSTRACTS 1.2

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Rearrangements and vortex rings

We study axisymmetric vortex rings without swirl, which are steady relative to a steadily translating frame, in an otherwise irrotational ideal fluid which occupies the whole of $\mathbb{R}^3$ and is stationary at infinity. We use the variational principle proposed by T.B. Benjamin. Here, kinetic energy is maximised over flows for which $\zeta := \omega/r$ (ω being the strength of the azimuthally-directed vorticity and r being the radius of cylindrical coordinates) is a rearrangement of a prescribed non-negative function ζ_0 having compact support, and for which the impulse in the axial direction has a prescribed value I. When $\zeta_0 \in L^p(\mathbb{R}^3)$ for some $p > 5/2$, we prove existence of the maximiser (which must represent a steady flow) in an extended constraint set which allows some loss of vorticity. We pay particular attention to the case when ζ_0 represents Hill's spherical vortex, whose impulse we denote I_0. When $I > I_0$ the maximisers are non-spherical and no loss of vorticity arises. If $I \leq I_0$ then the maximiser is spherical, and loss of vorticity occurs if the inequality is strict.