Impact of Market Design and Trading Network Structure on Market Efficiency

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Workshop on Complex Networks in Banking and Finance
Fields Institute in Toronto
June 25th, 2024

Presentation plan

1. Motivation: Chamberlin's (1948) experiments of market
2. Research Problem Definition:
   - Social welfare function & market efficiency
   - Zero-Intelligence Trading (ZIT) & Greedy Matching
3. Simulation results for:
   - complete bipartite graph
   - non-complete bipartite graph
4. Conclusions & Further Research

Motivation: new insights from experimental economics

Finance (and economics in general) - unlike natural science - is not well-suited to perform experiments like in laboratories on financial (economic) systems.

- The reason is that financial/economic costs of such experiments could be too costly for societies.
- As a result, most finance economists work with abstract/mathematical models to perform experiments on such math models.
- However, although macroeconomic experiments on the whole economy is prohibitive, microeconomic experiments have their track record.

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Motivation: Chamberlin's (1948) experiments of markets

Chamberlin (1948) conducted one of first experiments in finance/economics:

- By splitting his economics students (46 trials) into roughly equal-sized groups of: sellers (S) and buyers (B).
- Each seller i has one unit of good to sell at at minimum price of S_i, i.e. individual cost, e.g. a student receives a card S-18, meaning it's a seller role with $S_i = 18$.
- Analogously, each buyer j wants to buy one unit of good at the maximum price of B_j, called: willingness-to-pay (WTP), reservation price, redemption value, e.g. a student receives a card B-104, meaning it's a buyer role with $B_j = 104$
- Students interacted with each other to make a profitable transaction, i.e. negotiated such a transaction price p_{ij} between seller i and buyer j that $S_i \leq p_{ij} \leq B_j$, so nobody is losing and transaction is value-added.

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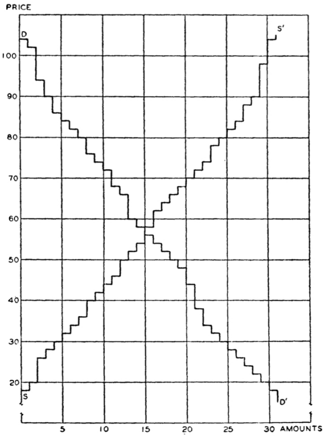
Definition of demand, supply, equilibrium price and volume

Perfect Competition model predicts equilibrium price and volume of transactions at the intersection of demand and supply curves:

- Demand function is constructed as a decreasing sequence of B_j
- Supply function is constructed as an increasing sequence of S_i
- Given all B_j and S_i a market organizer can calculate and set an **equilibrium price (56-58)** and **volume (15)**

Market Schedules

B	S
104	18
102	20
94	26
90	28
86	30
84	32
82	34
80	36
76	40
74	42
72	44
68	46
66	50
60	52
58	54
56	58
54	62
52	64
50	66
48	68
44	70
38	72
34	74
32	78
30	80
28	82
26	84
24	88
22	90
20	98
18	104



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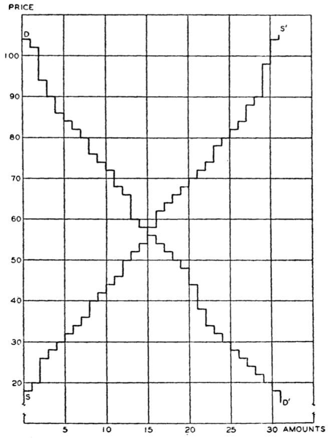

Fig. 1

Results of Chamberlin's (1948) experiments of markets

Objective of Chamberlin (1948) experiments:

Assessing the prediction accuracy of perfect competition model, i.e. if actual number of translation and prices is well predicted by a competitive model.

Results of Chamberlin (1948) experiments:

- Students "traded too much", i.e. the **actual volume (avg. 19) was higher than the equilibrium amount (15) 42 times out of 46 trials** and the same 4 times. It was **never lower**.
- Also an **actual price** (avg. 52.6) **diverged from equilibrium price (56 – 58)**

TRANSACTIONS

B	S	P
56	18	55
54	26	40
72	30	50
84	34	45
44	44	44
102	42	42
80	20	40
60	28	55
48	40	45
76	36	45
94	52	55
68	58	62
66	46	55
82	32	58
90	72	72
104	54	54
52	50	50
86	64	64
74	62	69

LEFT OVER

38	68
50	66
28	82
32	88
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Equilibrium sales 15
Actual sales 19

Equilibrium price 57 (56–58)
Average of actual prices ... 52.63

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Introducing the Social Welfare function...

- **Is "trading too much" a real problem?** More transactions sound like a good thing. Not necessarily for economists...
- ...since economists are more interested in **social welfare**, rather than quantity *per se*:

Definition of social welfare (SW)

Social welfare is a total value-added created from all trades, i.e. $SW = \sum_{\{(i,j):traded\}} B_j - S_i$

Definition of market efficiency

Efficiency of market is the fraction (between 0% and 100%) of social welfare achieved by this market out of maximum social welfare:

$$Eff = \frac{SW}{max_{(i,j)} SW} \quad (1)$$

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Research problem

Given two motivating observations demonstrated in the literature:

- varied degree of **market efficiency dependent on market designs**, e.g. perfect competition, Chamberlin (1948) Haggling, Market-clearing prices by Demange (1986), Double Auction in Smith (1962,64) and Gode&Sunder (1993)
- most of **social interactions** – including trading – **happen between a limited group of friends** or acquainted traders, see Jackson (2008), Easley & Kleinberg (2012), Newman (2018),

...we have put traders in a social (bipartite) network in order to investigate the interaction between two drivers of market efficiency:

- **market design**, i.e. Chamberlin's higgling market, greedy matching, Hungarian algorithm, perfect competition.
- **social network characteristics**, i.e. network density and size.

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Zero-Intelligence Trading market design definition

We define the **Zero-Intelligence Trading**[1] (**ZIT**) market design as a following simulation process described by Chamberlin in 1933:

1. Pick uniformly a random pair of cards for seller $i \in S$ and buyer $j \in B$
2. The pair (i, j) trades, if $S_i \leq B_j$
3. Iterate the process by picking uniformly a random pair of a seller and a buyer, who haven't yet traded, until no further trade is possible, i.e. $min_i(S_i) > max_j(B_j)$

For non-complete graphs, i.e. $(B \cup S, E_p)$ with $p < 1$, we pick uniformly random pairs that belong from edge set, i.e. $(i, j) \in E_p$.

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Greedy matching market design definition

ZIT will be bench-marked against **Greedy matching**, which is a following **deterministic** and **sub-optimal** process of matching sellers with buyers:

1. Initiate the set of trades T to $\emptyset$,
2. for each edge $(i^*, j^*) \in E_p$ calculate the value $SW(\{(i^*, j^*)\}) = B_{j^*} - S_{j^*}$,
3. sort the pairs (i^*, j^*) of sellers and buyers with respect to $SW(\{(i^*, j^*)\})$, in a non-increasing order,
4. iterate over sorted pairs (i^*, j^*):
    1. if neither of them has traded, i.e. $i^* \in \{i : \forall_j (i, j) \notin T\}$ and $j^* \in \{j : \forall_i (i, j) \notin T\}$ as well as $S_i \leq B_j$ then make the pair (i^*, j^*) trade and update the set: $T \leftarrow T \cup \{(i^*, j^*)\}$

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Simulation experiment setup

Simulation loops:

for *sim* $\in \{1, ..., 1000\}$ **do**

for $n \in \{10, 100, 1000\}$, $p \in \{0.001, 0.01, 0.01, 1\}$ **do**

Generate Random Bipartite Graph $G = (B \cup S, E_p)$

for $marketDesign \in \{Zero - IntelligenceTrading, greedy\}$ **do**

Run Trading process according to *marketDesign* on graph G

Calculate market efficiency (Eff) and trade participation

Where:

- *sim* - simulation iterator,
- n - number of sellers/buyers, network size is $2 \times n$,
- p - probability of an edge, hence degree is $n \times p$,
- E_p - set of edges between sellers and buyers, which is constructed in a way that for each pair of nodes $(i, j) \in S \times B = \{1, .., n\} \times \{1, .., n\}$, we independently introduce an edge (i, j) in E_p with probability p.

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Numerical results for ZIT on complete graphs, i.e. $p = 1$

We conducted simulation experiments of ZIT as well as employed differential equitation to demonstrate the following results on complete graphs, i.e. $p = 1$:

Numerical results for a model specification of: $S_i, B_j \sim iid\ U(0,1)$

- $Eff^{ZIT} \approx 73\%$, i.e. $\approx 27\%$ **value-destroyed**
- $\approx 71\%$ (instead of 50%) of traders do trade. Among them:
  - 50 p.p. are intramarginal traders, i.e.: $(\{i : S_i \leq \frac{1}{2}\}, \{j : B_i \geq \frac{1}{2}\})$
  - remaining $\approx 21\%$ p.p. are **extramarginal traders**, i.e. sellers $\{i : S_i > \frac{1}{2}\}$ and buyers $\{j : B_j < \frac{1}{2}\}$ - **too much trade**

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  - 50 p.p. are intramarginal traders, i.e.: $(\{i : S_i \le \frac{1}{2}\}, \{j : B_i \ge \frac{1}{2}\})$
  - remaining $\approx 21\%$ p.p. are **extramarginal traders**, i.e. sellers $\{i : S_i > \frac{1}{2}\}$ and buyers $\{j : B_j < \frac{1}{2}\}$ - **too much trade**

Numerical results for ZIT on complete graphs, i.e. $p = 1$

We conducted simulation experiments of ZIT as well as employed differential equitation to demonstrate the following results on complete graphs, i.e. $p = 1$:

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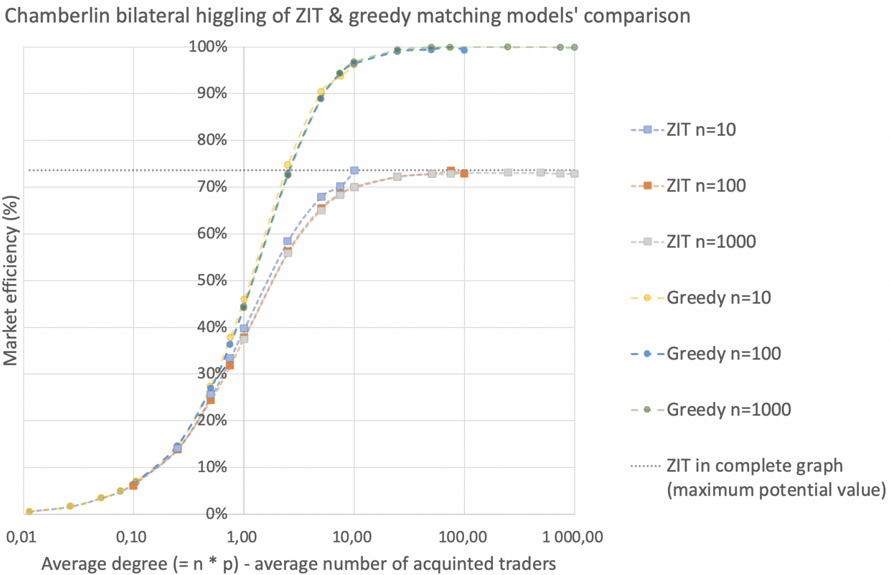
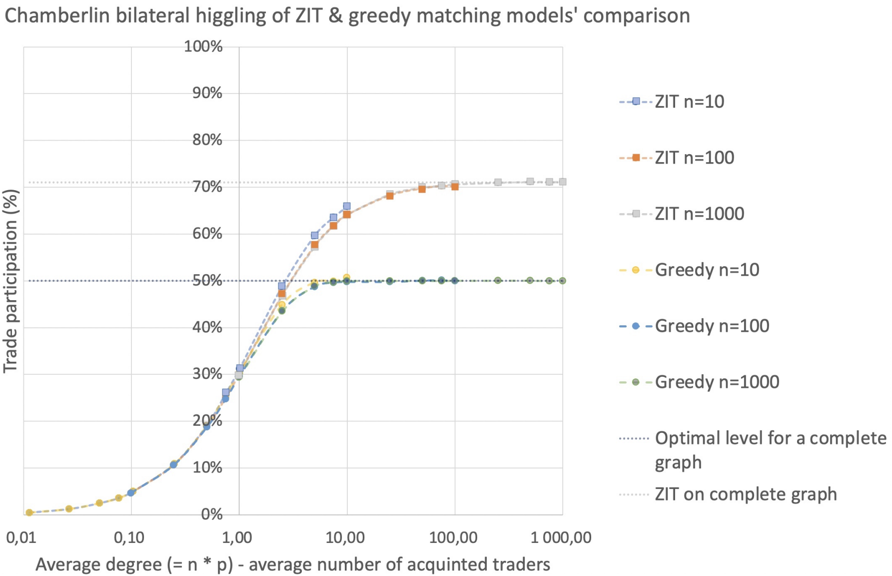
Simulated market efficiency for non-complete graphs $p \leq 1$



Simulated Participation rate for non-complete graphs $p \leq 1$



Conclusions

Simulations showed that:

- **For sparse graphs** of degree lower than 1, i.e. $n \times p \leq 1$, **both market design are comparable** its efficiency is ca. 40%-45% and converges to 0% as graph gets sparse.
- **Greedy matching outperforms ZIT significantly for non-sparse** ($n \times p \geq 1$) **graphs** and **converges to** 100% as graph gets denser.
- Market **efficiency** can be significantly **improved** for both market designs **by increasing the average degree (**$n \times p$**) from** 1 **to** 5 **or** 10, enabling greedy matching to improve from ca. 45% to 89% and 95%, respectively. **Practical take-away:**
  - It's enough to have **at least** 5 **friends to achieve ca.** 90% **of maximum efficiency** of either market design.
- Contrary to popular belief, **market size had no significant impact**.

Thank you for your attention!

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APPENDIX

Unpublished asymptotic results of further research (1/4)

Definition (Hungarian Algorithm Market Efficiency on $\mathcal{G}(n, n, p)$)

Let $H(p)$ be the market efficiency of Hungarian Algorithm on a random bipartite graph $\mathcal{G}(n, n, p)$ with vertex bi-partition $V = (S, B)$, $S = [n]$, $B = [n]$, in which each of the n^2 possible edges appears independently with probability p.

Theorem (On the market efficiency of Hungarian Algorithm)

Let $c = c(n)$ be any function of n, and let $p = \frac{2(\log n + c)}{n}$. Consider the process ran on $\mathcal{G}(n, n, p)$. Then,

$$\mathbb{P}\Big(H(p) = H(1)\Big) \to \begin{cases} 0 & \text{if } c \to -\infty \\ e^{-e^{-\hat{c}}} & \text{if } c \to \hat{c} \in \mathbb{R} \\ 1 & \text{if } c \to \infty. \end{cases}$$

Hungarian Algo efficiency does not change unless the graph is very sparse.

Unpublished asymptotic results of further research (2/4)

Let $G(p)$ denote the market efficiency generated by greedy matching. Trivially, $G(p) \leq H(p)$. Following theorem shows that a.a.s. the greedy algorithm ran on $\mathcal{G}(n,n,p)$ is asymptotically as good as the optimal Hungarian Algorithm ran on the complete bipartite graph, provided that the expected degree tends to infinity, that is, $np \to \infty$ as $n \to \infty$.

Theorem (On the market efficiency of Greedy Matching)

Let $\omega = \omega(n) = \mathcal{O}(\log n)$ be any function of n that tends to infinity (sufficiently slowly) as $n \to \infty$. Let

$$\hat{\omega} = \hat{\omega}(n) = \frac{\omega}{3\log\omega} = \mathcal{O}\left(\frac{\log n}{\log\log n}\right) \qquad \textit{and} \qquad p = p(n) = \frac{\omega}{n}.$$

Consider the process ran on $\mathcal{G}(n,n,p)$. Then, a.a.s.

$$G(p) \geq \frac{n}{4} + \mathcal{O}\left(\frac{n}{\hat{\omega}}\right) \sim \frac{n}{4} \sim H(1).$$

Unpublished analytical results of further research (3/4)

Definition (Full greedy matching)

Let the full greedy matching process be equivalent to greedy matching as **defined before on slide page 12**.

Definition (Sequential greedy matching)

Let the sequential greedy matching be the **sequential random selection of an active side followed by greedy selection of its member and greedy selection of this member neighbour**.

Theorem (On the equivalence of full and sequential greedy matching)

Both *full and sequential greedy matching processes* ***produce the same list of trades****.*

Further unpublished simulation results (4/4)

- The market efficiency ratio of both ZIT/random and greedy matching to Hungarian Algorithm is **U-shaped wrt the average degree**,
- Conditional positive impact of the **imbalance btw demand& supply**:
  - The **imbalance** between demand & supply measured as the deviance of probability of choosing a buyer as an active side from 50%, i.e. $|buyerPr - 50\%|$, **doesn't impact the efficiency, if there's only one auctioneer and an inactive player is chosen in a random manner**,
  - The **imbalance** between demand & supply, as defined above, **has a positive impact on market efficiency, if the active player is chosen in a greedy manner or there's at least 2 auctioneers**.
- Highly **conditional impact of single search**:
  - The **impact of single search** is **positive for at least** 2 **auctioneers** and **is decreasing with the number of auctioneer**
  - **For** 1 **auctioneer the impact of single search is**:
    - **negative for average degree higher than or equal to** ≈ 10, i.e. $n_s \times p = n_b \times p \geq\approx 10$
    - **positive for average degree lower than or equal to** ≈ 3, i.e. $n_s \times p = n_b \times p \leq\approx 3$

Differential equation approach

Define the function $X : \mathbb{R}_+ \times [0,1] \to [0,1]$ by the following differential equation:

$$\frac{\partial}{\partial t} X(t,v) = -X(t,v) \int_0^{1-v} X(t,z)dz.$$
$$X(0,v) = 1 \text{ for } v \in [0,1].$$

By monotonicity, it is clear that $\bar{X}(v) = \lim_{t\to\infty} X(t,v)$ exists.

I claim that $\bar{X}(v)$ gives the limiting probability (as $n \to \infty$) that a seller with value v does not trade, which is also the probability that a buyer with value $1-v$ does not trade.

The total fraction of participants who trade is given by

$$T = 1 - \int_0^1 \bar{X}(v)dv,$$

and the total revenue pocketed by the intermediary is

$$\Pi = \int_0^1 \bar{X}(v)(2v-1)dv.$$

What is the value of these quantities? It seems that $\bar{X}(v) = 0$ for $v \leq 1/2$, and $\bar{X}(v) \geq 2v-1$ for $v \geq 1/2$ (the actual function has some curvature on the interval $[1/2, 1]$). From these bounds we get that $\Pi \geq \int_0^1 \max(2v-1,0)^2 dv = 1/6$, whereas the optimal offline revenue is $1/4$.

I don't yet know how to solve the differential equation exactly to get an analytical expression for $\bar{X}(t)$, but numerically, it seems that $T \approx 0.71$ (71% of participants trade), and $\Pi \approx 0.184$ (which is approximately 73% of the optimal revenue).