Netting and Novation in Repo Networks with Rehypothecation: an Agent-Based Computational Model

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- Under re-hypothecation, B can use the asset A^R in order to borrow cash from another lender C.

Two examples of repo transactions

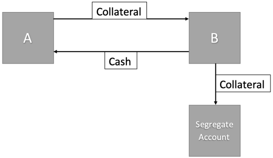


Figure: Simple repo

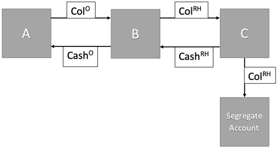


Figure: Repo with rehypothication

The balance sheet of bank i

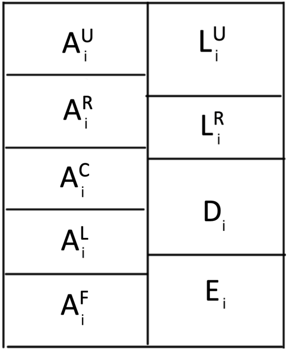


Figure: Assets: unsecured interbank loans A_i^U, reverse-repo (secured) loans A_i^R to other banks, general collateral A_i^C, liquid assets A_i^L and fixed assets A_i^F. Liabilities: unsecured interbank loans L_i^U and repo (secured) loans L_i^R from other banks, deposits D_i and equity E_i.

The network

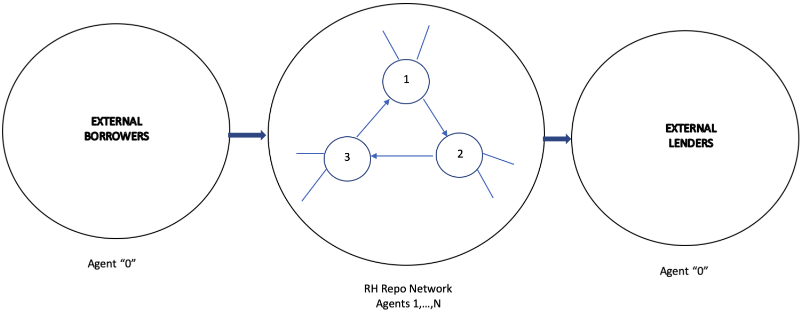


Figure: Banks and labelled by $\{1, 2, \ldots, N\}$ and all external agents are collectively labelled by "0", with edges given by nonzero entries of the matrices A^U_{ij} and A^R_{ij}.

The Gai-Haldane-Kapadia (2011) framework

- GHK (2011) consider the balance sheet and network as above and a liquidity constraint of the form

$$A_i^L+(1-h-h_i)A_i^C+\frac{(1-h_R-h_i)}{(1-h)}A_i^R+L_i^N-L_i^R-\lambda\mu_i L_i^U-\varepsilon_i>0.$$

- If this is violated by bank i, they assume that it "hoards" liquidity by recalling a fraction of A_i^U, thereby contributing to the violation of this constraints for other banks that borrowed from it.
- Because the effect of each shock of the form $-\lambda\mu_i L_i^U$ tends to be smaller the higher the number of counterparties, of i, one obtains a "tipping point" behaviour, with network with average connectivity below a threshold z^* experiencing "contagion", in the sense that an initial liquidity shock would spread to all banks.

Funding sources and balance sheet constraints

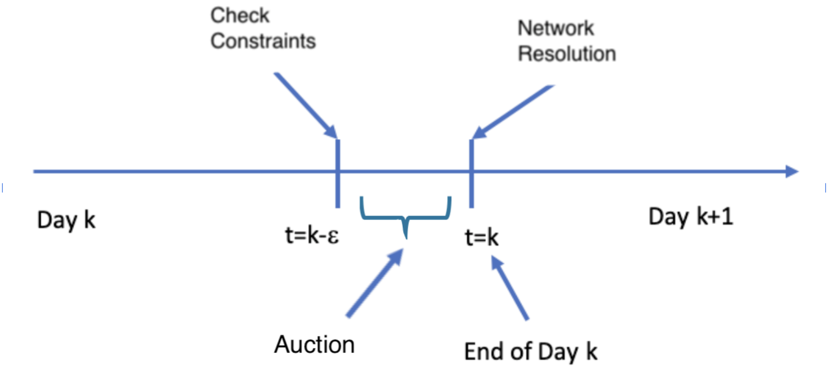
- We extend the GHK framework by allowing banks to raise funds either by selling illiquid fixed assets A^F (see α below) or recalling of reverse-repos A^R.
- We assume for simplicity that $A^U = 0$.
- Accordingly, we consider three separate balance sheet constraints:

$$\mathcal{C} := (1-h)A_i^C + \frac{1-h_R}{1-h}A_i^R - L_i^R \geq 0 \qquad \text{(collateral)} \quad (1)$$

$$\mathcal{L} := A_i^L \geq 0 \qquad \text{(liquidity)} \quad (2)$$

$$\mathcal{E} := E_i \geq 0 \qquad \text{(solvency)} \quad (3)$$

Timeline


Check
Constraints
Network
Resolution
Day k
t=k-ε
t=k
Day k+1
Auction
End of Day k

Auction

- Let $\mathcal{N}_k^a$ be the subset of solvent banks (i.e satisfying (3)) that are either illiquid or insufficient collateralized
- Because general collateral is highly liquid *and* needed to guarantee secured loans, we assume that the auction begins with a **collateral sub-step** when banks attempt to either restore (2) by selling excess collateral or restore (1) by selling fixed assets. Banks that fail the latter are removed.
- This is followed by a **liquidity sub-step**, where remaining banks attempt to restore (2) either by selling A^F or recalling A^R.

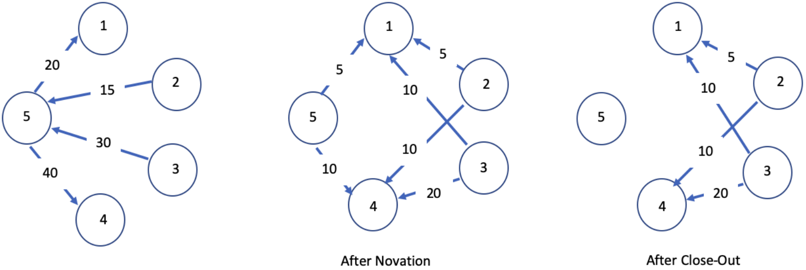
Resolution

- Removing defaulted banks consist of two steps: netting and novation.
- In the **netting step**, we remove all cycles in the resolution subset $\sigma = \{i_1, \ldots, i_m\} \subset \{1, \ldots, N\}$, meaning that the resulting graph satisfies

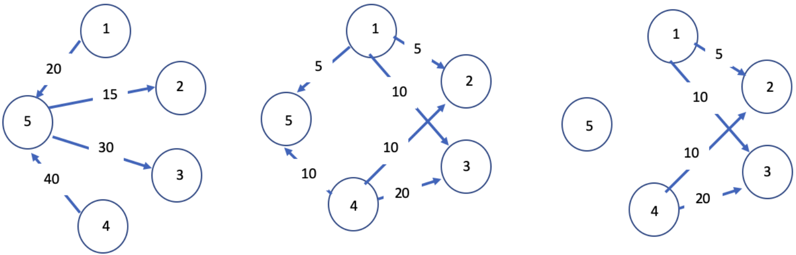
$$L_{i_\ell i_1} \prod_{k=2}^{\ell} L_{i_{k-1} i_k} = 0,$$

  for any set of nodes in σ.
- In the **novation step**, assets and liabilities of the removed banks are rewired proportionally to all remaining banks.

Novation of a net-borrowing bank



Novation of a net-lending bank



After Novation

After Close-Out

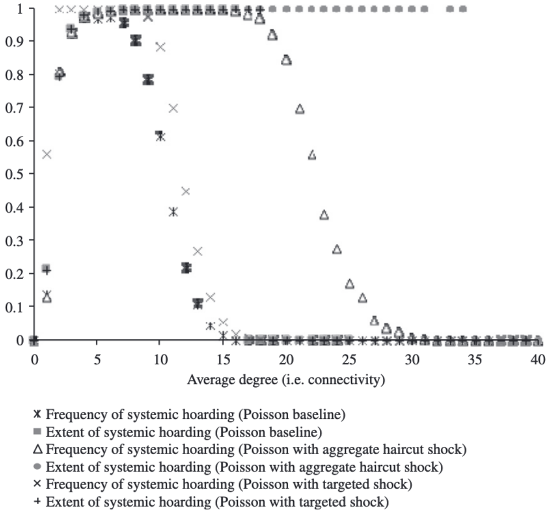
Analytical Results

- After novation of a single bank, it becomes either a pure lender or a pure borrower, and the assets and liabilities of all other banks remain unchanged.
- If $L_{ij}L_{ji} = 0$, novation of i and j is order-indifferent.
- If $\sigma = \{i_1, \ldots, i_m\} \subset \{1, \ldots, N\}$ is acyclic, then it remains acyclic after novation of any bank in σ.
- Therefore, after the netting step, multi-bank novation can be uniquely defined as successive single-bank novation in any order.

Numerical Results

- We use $N = 250$, $A_i^L = 20$, $E_i = 50$, $A_i^R = 300$ for all banks.
- In each experiment we use Poisson and geometric distribution of links to create unweighted networks and then distribute the total assets A_i^R equally among the counterparties of bank i.
- Deposits D_i and fixed assets A_i^F are then obtained as residuals.
- For each experiment we considered 3,000 simulations and report the following:
  - Systemic hoarding frequency (SHF) : fraction of the simulations with at least 10% of banks recalling reverse-repos.
  - Systemic hoarding extent (SHE): proportion of banks recalling reports conditioned on systemic hoarding.
  - Systemic default frequency (SDF) : fraction of the simulations with at least 10% of banks defaulting.
  - Systemic default extent (SDE): proportion of banks defaulting conditioned on systemic default.

GHK - Experiment 1

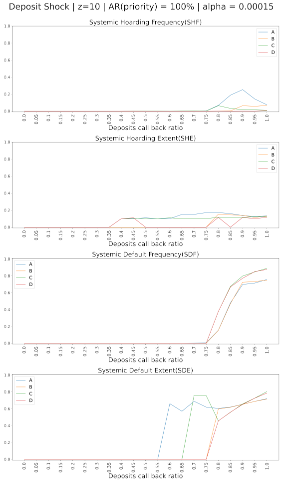


$\Delta D_i = -D_i$, $\alpha = 0.00075$

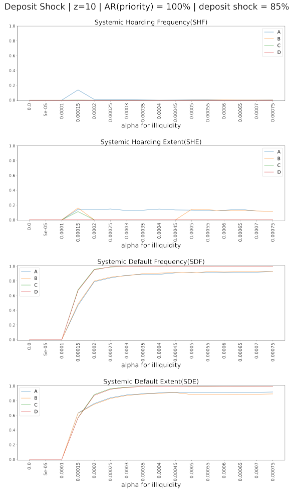
Deposit Shock

$\Delta D_i = -D_i$, $\alpha = 0$

Varying ΔD_i, $\alpha = 0.00015$



$\Delta D_i = -0.85 D_i$, varying α



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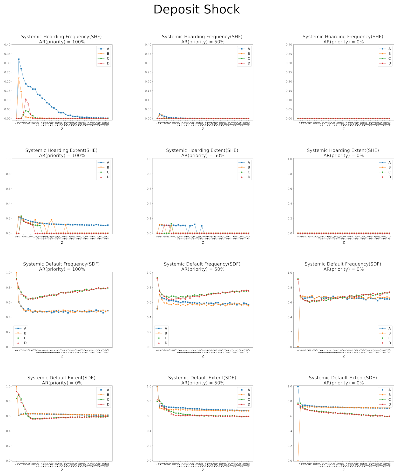

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- We use the proposed model to investigate the behaviour of the network through a number of numerical experiments.
- Systemic default is by far the dominant channel of shock propagation whenever α is non-negligible.
- Specific combinations of α and size of an initial liquidity shock do, however, have the capacity to generate both systemic default and systemic hoarding of liquidity in the repo market.