# Decomposing Systemic Risk: The roles of Contagion and Common Exposures

## A Structural Approach

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*The views expressed in this paper are solely those of the authors and may differ from official BoC or ECB views.*

# Some codifying semantics

**Systemic risk** Threat of impairment of the financial system via correlation of distress[1]

- e.g., GFC

**Systematic risk** Non-diversifiable risk as a threat to the financial system

- from common exposures/portfolio overlaps

**Contagion** An initial idiosyncratic problem getting more widespread to the system

- cross-holdings (network effects), price mediated effects (fire sales), funding-risk spillovers (sentiment-based)

Loosely, we decompose:
Systemic risk = Contagion*(Systematic risk + Idiosyncratic Risk)

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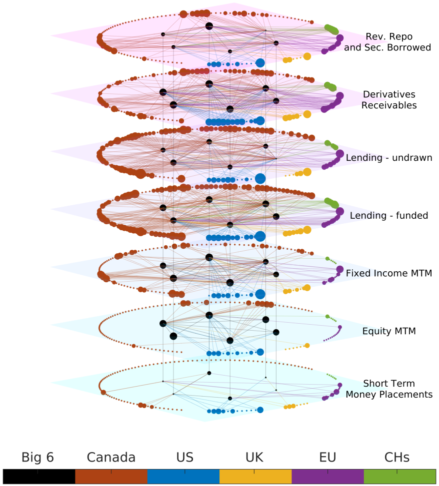
# Financial interconnectedness: directed weighted networks

Financial interconnectedness makes shocks propagate through the system.

But networks may have different sensitivities per layer.

How can we quantify these effects?




Multilayer Network for Canadian banks. Nodes represent FIs, arrows connections, layers asset classes, and colors FIs-categories. Source: EBET-2A

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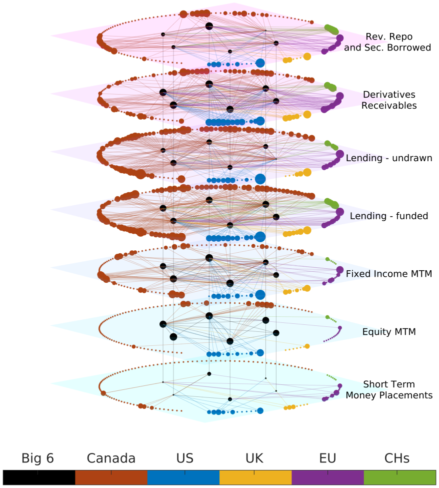


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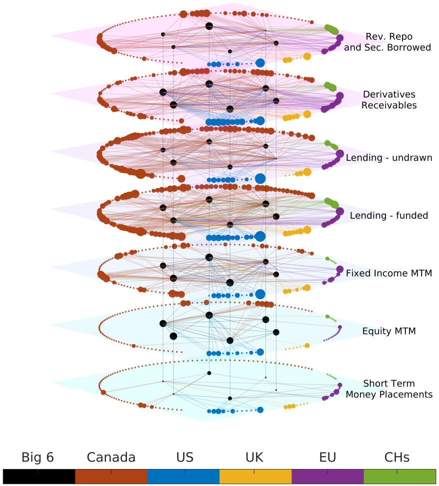


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## Research Question(s)

1) How do we quantify contagion/spillovers?
2) How much do different systemic risk channels contribute to the transmission of shocks between banks?

## Method

Estimation: Use correlation of time series.
Identification: the observed network provides equality constraints.

# Model

A multilayer network regression.

# Contemporaneous dependency

The systems' observations in a vector $y_t$. Then there are only two sources of variations: endogenous and exogeneous

$$y_t = \underbrace{\sum_a \beta_G^a G^a \, y_t}_{endogenous} + \underbrace{X\varepsilon_t}_{exogeneous} \qquad \forall t$$

- $y_t$ $(N \times 1)$ vector of observables (here capital ratios of banks)
- $G^a$ observed zero-diagonal network matrices for $a = 1, ..., A$
- $\beta_G^a$ network sensitivity
- $X$ observed dimension reducing matrix $(N \times S)$
- $\varepsilon_t$ $(S \times 1)$ exogenous vector, $\varepsilon_t \sim \mathcal{N}(0, \Sigma_\varepsilon)$

BalanceSheetIdentity

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# Solving for $y_t$

$$\left(I_N - \sum_a \beta_G^a G^a\right) y_t = X \cdot \varepsilon_t, \tag{1}$$

$$y_t = \underbrace{\left(I_N - \sum_a \beta_G^a G^a\right)^{-1}}_{Leontief\ Inverse} \cdot X \cdot \varepsilon_t, \qquad \text{Var}\left[\varepsilon_t\right] = \Sigma_\varepsilon$$

Then, the covariance matrix reads

$$\text{Var}[y_t] = \left(I_N - \sum_a G^a \beta_G^a\right)^{-1} X \Sigma_\varepsilon X' \left(I_N - \sum_a G^a \beta_G^a\right)^{-1\,\prime},$$

This we can estimate, and knowledge about $G^a$ identifies $\beta$ and $\Sigma_\varepsilon$. ▸ Identification

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# Application

Data:

- Monthly data from 2007 to today.
- 6 big and 4 smaller banks in Canada.
- Missing data is interpolated and returns are extrapolated for length.

Contagion via

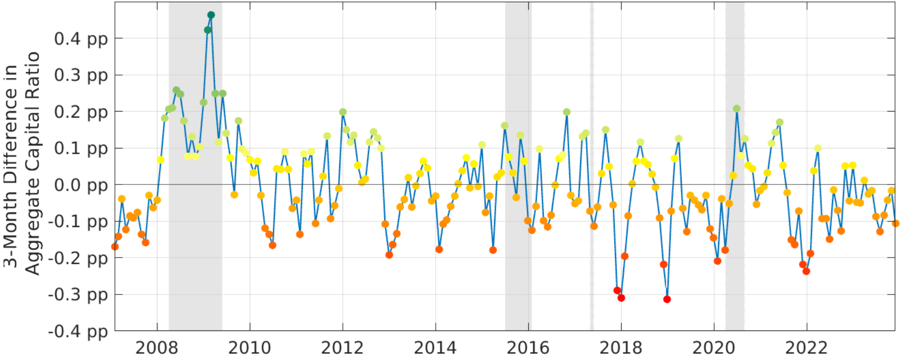
- six inter-bank asset categories, $(G^a)$,
- investor-sentiment networks (Hipp (2020) similar to Diebold and Yılmaz (2014)),
- price-mediated effects (e.g., firesales) through common holdings.

Exogenous impacts via mortgage loans, business loans, securities, and funding, $(X)$.

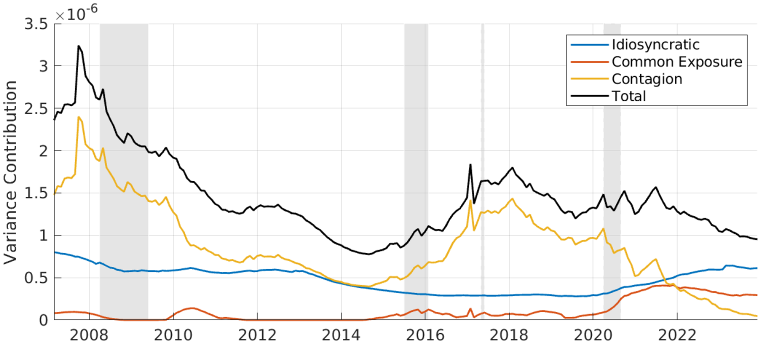
# Was there any stress to decompose?

Index created by differences in system capital ratio.

$$Index = \Delta Y_t, \qquad Y_t = w' y_t.$$ (System capital ratio)

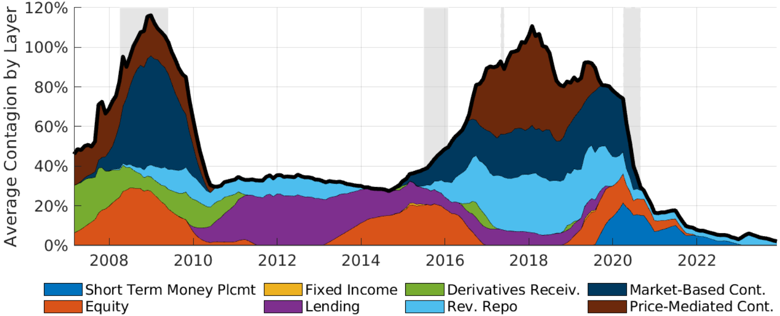


# System's Variance Decomposition by Channel



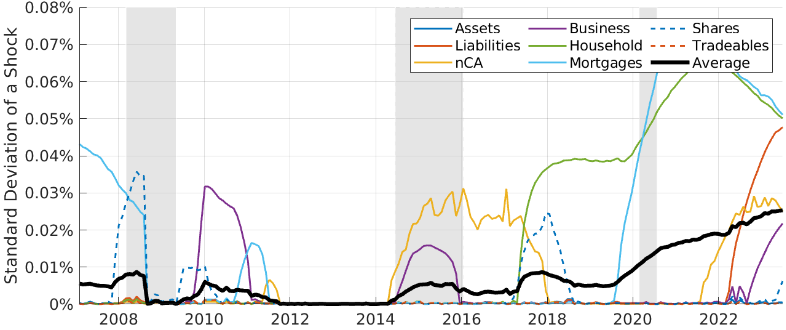

- High variance during GFC
- Strongest contribution from contagion
- idiosyncratic channels lower after Basel III, but increase with common exposures after COVID

# Decomposing Contagion




- Market-based and price-mediated contagion mostly responsible for peaks.
- Low spillovers/network effects today.

# Which shocks contributed the most?




- During the GFC, housing and share shocks are more prominent.
- Lately, housing, foreign and liabilities.

## Conclusion

- We developed a novel method to quantify contagion.
- We decomposed systemic risk in different channels over time.
- Contagion is in general important, but lately banks common exposures contribute more risk.

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# Literature I


Paolo Barucca, Marco Bardoscia, Fabio Caccioli, Marco D'Errico, Gabriele Visentin, Guido Caldarelli, and Stefano Battiston. Network valuation in financial systems. *Mathematical Finance*, 30(4):1181–1204, 2020.

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Francis X Diebold and Kamil Yılmaz. On the network topology of variance decompositions: Measuring the connectedness of financial firms. *Journal of Econometrics*, 182(1):119–134, 2014.

Ruben Hipp. On causal networks of financial firms: Structural identification via non-parametric heteroskedasticity. *Bank of Canada Staff Working Paper Series*, 2020.

Thomas J. Rothenberg. Identification in parametric models. *Econometrica: Journal of the Econometric Society*, pages 577–591, 1971.

# The valuation vector $v_k$

For example, take a lending between bank $i$ and $j$

$$v_j^l(\lambda, \varepsilon) = 1 + f^l(\lambda_j) + \varepsilon$$
$$\frac{\partial f^l}{\partial \lambda_j} = \beta^l$$

- The valuation depends on the leverage ratio of bank $j$, $\lambda_j$, and exogenous shocks $\varepsilon$
- Endogeneity will follow from valuation-sensitivities $\beta_k$

back

# More granularity, endogeneity, and normalization

$$\lambda = \sum_a G^a \cdot v_G^a(\lambda, \varepsilon_G^a) + A^{ex} \cdot v_A(\lambda, \varepsilon_A) - L \cdot v_L(\lambda, \varepsilon_L) + \$,$$
$$\lambda = F(\lambda, \varepsilon).$$

$G^a$ interbank network for asset class $a$

$\lambda$ leverage ratio

$\varepsilon$ exogenous shock vectors

▸ Valuation Vector

# Making it dynamic

## Decomposition equation – four channels

Taking the total differential and equip with time index $t$:

$$y_t = \underbrace{\sum_a G^a \beta^a_{G,t}\, y_t}_{(I)} + \underbrace{\widetilde{A}^{ex}_t \beta_{A,t}}_{(II)} y_t - \underbrace{\widetilde{L}_t \beta_{L,t}}_{(III)} y_t + \underbrace{[\widetilde{A}^{ex}_t, \quad -\widetilde{L}_t] \cdot (\varepsilon_A, \quad \varepsilon_L)'_t}_{(IV)}.$$

(I) Contagion via direct interbank exposures ▸ interbank

(II) Price-mediated contagion ▸ Price-mediated

(III) Contagion via market-based networks ▸ Market-based

(IV) Common exposures ▸ Common exposures

▸ NEVA

# Channel (I) – multilayer-network

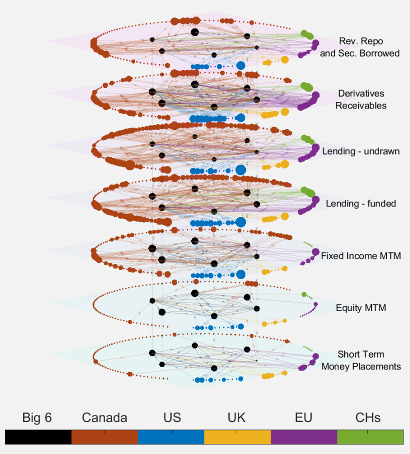



Figure: Multilayer network. Each layer represents an asset class and thus can face a different contagion coefficient. back

# Channel (II) - Price-mediated contagion

$$\frac{\partial(\widetilde{A}^{ex}\beta_A(\lambda)d\lambda)}{\partial\gamma} = A \cdot A' \cdot \text{diag}(\lambda)^{-1} d\lambda. \qquad (N \times N)$$

$\gamma$ – price-impact function, i.e. sensitivity of valuation to aggregate volume of rebalanced assets Quadratic impact of changes in banks' portfolios on earnings -> capital -> leverage

▸ back

# Channel (III) – Market-based

Funding pressures from external creditors

$$f_L(e) = (\underbrace{I_N}_{idiosyncrasies} + \overbrace{G^{MB}}^{market-based})e \otimes \underbrace{\overrightarrow{a}}_{deposit-sensitivities}, \tag{2}$$

$\overrightarrow{a}$ is an $(K^L \times 1)$ vector of deposit-sensitivities and $G^{MB}$ is the investor anticipated network of spillovers with zeros on the diagonal.

Example: Assume bank A gets distressed and thus faces higher funding costs on the market. Now, if investors anticipate bank B to be connected to A, they assume it is distressed too and thus B faces higher funding costs too. This can happen in the absence of real economic links.

back

# Channel (IV) – Common Exposures

$$d\lambda = [\widetilde{A}, \quad -\widetilde{L}] \cdot \Theta \cdot \mathrm{diag}(\beta^C)\varepsilon = [A, \quad -L] \cdot \Theta \cdot \varepsilon_C, \tag{3}$$

where $\varepsilon_{\mathcal{C}}$ is the vector of all systematic shocks, and $\Theta$ the selection matrix resembling $\mathbf{1}(k \in C)$, i.e., a $(K + N * H) \times \#\mathcal{C}$ matrix that contains a $1$ in position $kC$ if the $k$th balance sheet item is affected by the systematic risk $C$ and $0$ else.

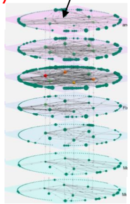
back

# Small example of contagion

$$\begin{pmatrix} d\lambda_{CIBC} \\ \vdots \\ d\lambda_{NBC} \end{pmatrix}_t = \underbrace{\begin{pmatrix} \beta_1 \\ \vdots \\ \beta_7 \end{pmatrix} * \boldsymbol{G} * \begin{pmatrix} d\lambda_{CIBC} \\ \vdots \\ d\lambda_{NBC} \end{pmatrix}_t}_{\text{Contagion}} + \underbrace{[\boldsymbol{A}, -\boldsymbol{L}] * \begin{pmatrix} B_1 \varepsilon_1^{mkt} \\ \vdots \\ B_9 \varepsilon_9^{mkt} \end{pmatrix}_t}_{\text{Common Exposures}} + \underbrace{\begin{pmatrix} B_{CIBC} \varepsilon_{CIBC} \\ \vdots \\ B_{NBC} \varepsilon_{NBC} \end{pmatrix}_t}_{\text{Idiosyncratic risks}}$$

**change in leverage ratio**

**Contagion**



*New granular network data from EBET-2A*

**Common Exposures**

Portfolio Composition matrix

| Securities | Ca Deposits |
| --- | --- |
| Mortgages | Foreign Dep. |
| ⋮ | ⋮ |

*Loans (A2), Securities (B2), Mortgages (E2)*

**Idiosyncratic risks**

*Shocks in yellow structurally identified*

*Parameters in red are estimated*

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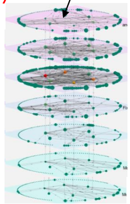

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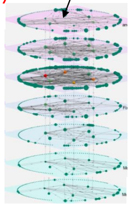

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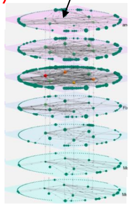

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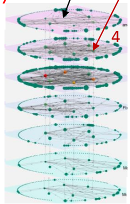

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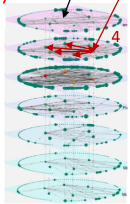

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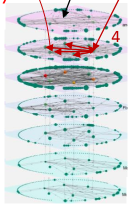

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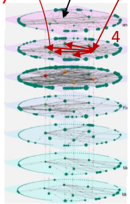

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| ⋮ | ⋮ |

*Loans (A2), Securities (B2), Mortgages (E2)*

**Idiosyncratic risks**

*Shocks in yellow structurally identified*

*Parameters in red are estimated*

# Extension and estimation of Network Valuation of exposures

Barucca et al. (2020)

Original NEVA:

$$e_i = A_i^{ex} - L_i^{ex} + \sum_{j=1}^{N} a_{ij} \mathbb{V}([e_1, \ldots, e_N]) - \sum_{j=1}^{N} l_{ij}$$

What we change to estimate the model?

- multi-layer network of exposures (rather then 1-layer)
- valuation of liabilities (rather than contractual)
- impact of changes in ill-liquid asset classes, ie price-mediated contagion (rather than indirect contagion only through exogenous valuation of direct exposures)

back

# Covariance and Correlation

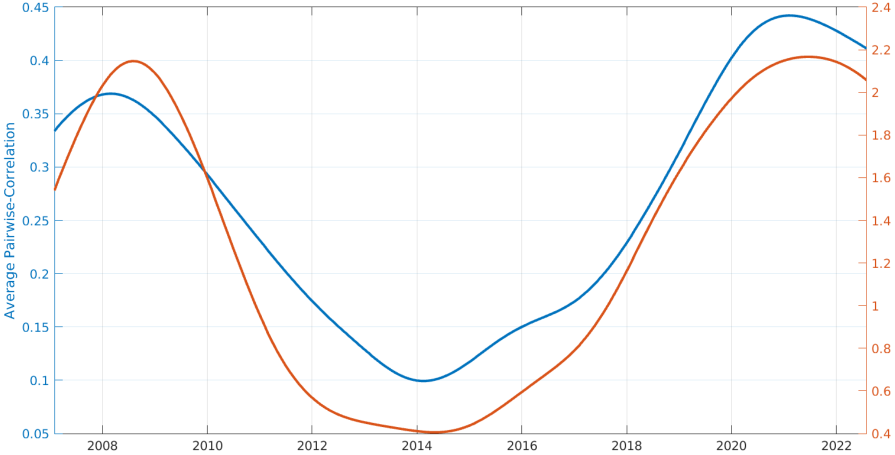


Figure: Pairwise covariance and correlation over time.

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# Start: Balance sheet identity

$$e = Av_A - Lv_L, \qquad (4)$$

- $e$ $(N \times 1)$ vector of equity positions of banks
- $A$ matrix of assets
- $L$ matrix of liabilities
- $v_{A,L}$ valuation vector (normalized to one)

▸ econometricEquation

# Metric Calculations

$$y_t = \underbrace{\left(I_N - \sum_a G^a \beta_G^a - \widetilde{A}_t \beta_A - \widetilde{L}_t \beta_L\right)^{-1}}_{Leontief\ Inverse:\ Contagion} \cdot \left(\underbrace{[\widetilde{A}_t, \quad -\widetilde{L}_t] \cdot \varepsilon_t^M + \varepsilon_t^I}_{Systematic\ and\ Idiosyncratic}\right) = \mathcal{A}_t^{-1}\mathcal{B}$$

Take $(N \times N)$ contagion matrix as $Con_t = \mathcal{A}_t^{-1} - I_N$, with average contagion:

$$average\ contagion = \frac{1}{N}\sum_i \sum_j Con_{ij,t}$$

.

Take $(N \times K)$ common exposure matrix as $Comm_t = [\widetilde{A}_t, \quad -\widetilde{L}_t]\text{diag}(\sigma_\varepsilon^M)^{0.5}$ , with average common exposure:

$$average\ contagion = \frac{1}{N}\sum_i \sum_k Comm_{ik,t}$$

Define the $(N \times N)$ idiosyncratic risk matrix as $Idio_t = \text{diag}(\sigma_\varepsilon^I)^0.5$ , with average common exposure:

$$average\ idiosyncratic = \frac{1}{N}\sum_i \sum_j Idio_{ij,t}$$

# Identification

## Sketch

The mapping from "observed/estimated" to "unknown/to be identified parameters" reads

$$\text{Var}[y_t] = \overset{\swarrow N(N+1)/2 \ equations}{\Sigma} = \underbrace{\mathcal{A}^{-1}(\beta)\mathcal{B}(\Sigma_\varepsilon)\ \mathcal{A}^{-1}(\beta)\ '}_{S(S+1)/2+A \ unknowns}$$

If $S(S+1)/2 + A < N(N+1)/2$ and no "multicollinearity" of networks, the Jacobians have full rank and Theorem 6 of Rothenberg (1971) identifies the mapping.

Intuitively, the observed network matrices impose equality constraints (as used in the SVAR literature).

Estimation via maximum likelihood or method of moments. back

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