On the rigidity of some Hirzebruch genera

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$\varphi([M]) = \langle \prod \frac{x_i}{f(x_i)}(\mathcal{T}M), [M]_{\mathbb{Z}} \rangle$

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The equivariant extension of a genus $\varphi\colon \Omega^*_U \to R$ is a composition

$$\varphi^T\colon \Omega^*_{U:T^k} \xrightarrow{\Phi} \Omega^*_U[[u_1,\dots,u_k]] \xrightarrow[u_i\mapsto f(x_i)]{\varphi\colon \Omega^*_U\to R} R[[x_1,\dots,x_k]]$$

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A genus $\varphi\colon \Omega_U^* \to R$ is rigid on M if and only if we have $\varphi(E) = \varphi(M)\varphi(B)$ for any fibre bundle $E \to B$ with fibre M.

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Theorem (Buchstaber–Panov–Ray localization formula)

If a T^k-manifold M has only isolated fixed points, then

$$\varphi^T(M) = \sum_{p \in M^T} \sigma(p) \prod_{i=1}^{n} \frac{1}{f(\langle w_i(p), \mathbf{x} \rangle)}$$

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  the elliptic genus is the universal $\mathbb{H}P^2$-rigid genus (Kreck–Stolz)

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Let φ be a genus which is rigid on S^6.
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In particular, $y_5 = [\widetilde{L}(2,3)]$.

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Corollary

The Krichever genus is the universal genus which is rigid on S^6 and $\widetilde{L}(2,3)$. In particular, it is the universal SU-rigid genus.

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Theorem

The Witten genus is the universal genus which is rigid and vanishes on $\mathbb{O}P^2$.

The rigidity equation for $\mathbb{O}P^2$ is equivalent to

$$\begin{aligned} 0 = f(y_1+y_2)f(y_1-y_2)f(y_3+y_4)f(y_3-y_4)+ \\ + f(y_2-y_3)f(y_2+y_3)f(y_1-y_4)f(y_1+y_4)+ \\ + f(y_2-y_4)f(y_2+y_4)f(y_3-y_1)f(y_1+y_3) \end{aligned}$$

Thank you for your attention!